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On the conductivity of antidot lattices in magnetic fields

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Abstract. We present results for the density of states, and the band conductivity, of strongly modulated lateral-surface superlattices in a magnetic field. The density of states with an energy smoothing of 0.4 meV shows roughly periodic oscillations of the density of states as the magnetic field is varied, comparable in magnitude and period to the oscillations seen experimentally in the resistance. The oscillations are found to correspond to the perturbed, periodic, cyclotron orbits. The band conductivity shows periodic oscillations with a period of one flux quantum per unit cell of the periodic potential related to Hofstadter's butterfly, but these are suppressed much more strongly than the density of states oscillations by inelastic scattering of the electrons.

1. Introduction

Recent results [1, 2] have shown the presence of quantum mechanical effects in the magnetoresistance of a 2DEG subject to a strong, periodic potential. By strong, we mean that the potential is sufficiently large for the higher parts of the potential to be above the Fermi energy of the electrons, producing roughly circular obstacles, referred to as antidots. The gross variations of the resistance in such systems have been explained in terms of classical mechanics [3, 4, 5, 6]. In this paper we are interested chiefly in the fine structure, on the scale of a change in magnetic field of the order one flux quanta per unit cell. This fine structure has been interpreted [1] in terms of variations in the density of states. Variations in the density of states modulate the scattering rate of the electrons, and so modulate the conductivity. The variations in the density of states have been related to the periodic orbits existing in the system.

In this paper we calculate results for the band conductivity and the density of states in the regime of the experiments, namely a lattice period of 300 nm, a Fermi energy for the electrons of around 10 meV and a magnetic field from 0 to 1.5 T. For a simplified, hard-wall, potential we find relatively simple results, with oscillations in the density of states in the region where the cyclotron radius is around half the lattice period appearing as a continuation of the Landau level structure at higher magnetic fields. For a softer potential we find variations of the density of states in this regime on a magnetic field scale consistent with the area enclosed by a (perturbed) cyclotron orbit, with similar oscillations continuing to higher magnetic fields. At low magnetic fields the band conductivity shows oscillations consistent with the presence of Hofstadter's butterfly.

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2. Model

With the assumptions of a fixed scattering time and a periodic potential, we calculate the longitudinal band conductivity using the formula described recently by Degani and Leburton [7]:

$$\sigma_{yy} = \frac{e^2\tau}{A}\sum_{k_y}\sum_{k_x}\sum_{n} v_y^2\left(-\frac{\partial f}{\partial E}\right) \tag{1}$$

where τ is the transport scattering time, v_x is the group velocity of the mode in the x direction, A is the area of the system, and f is the Fermi–Dirac distribution function. For simplicity we ignore spin splitting throughout this paper. The general validity and the method of solving equation (1) have been discussed in a previous paper [8] referred to below as paper I.

In the following we use a transport scattering time of $\tau = 38$ ps, corresponding to a mobility of $100\,\mathrm{m}^2/\mathrm{Vs}$. The quantum relaxation time—that is the time for an electron to be scattered out of a plane wave state—is typically about a tenth of the transport relaxation time because of the long-range nature of the Coulomb potential [9, 10], and we use values of $\tau_q = 6.5$ ps and $\tau_q = 0.81$ ps in the density of states calculations [11]

There is also a scattering (collisional) contribution to the conductivity in a magnetic field, which we do not attempt to calculate. We do, however, calculate the density of states, which is expected to affect the conductivity in two ways. Firstly, the density of states affects the inelastic scattering time, and so the scattering conductivity. Secondly, the density of states affects the screening in the 2DEG, and so the periodic potential, and the random potential from the ionized donors. Since we are not able to do a self-consistent calculation of these effects we present here simply results for the density of states, and do not attempt to translate these into variations in the conductivity. In so far as the variations in the density of states are small, we expect the consequent fractional variation in the conductivity to have a similar order of magnitude to the fractional variation in the density of states. For the parameters used here, we have shown [12] that the scattering conductivity is a small part of the total conductivity, although, as we find below, variations in the scattering conductivity are more robust in the presence of disorder and inelastic scattering than are variations in the band conductivity.

The calculations are done including an imaginary component to the energy, which is necessary for the stability of the calculation, and represents an effective scattering time, $\tau = \hbar/(2\ \mathrm{Im}(E))$. With the imaginary component of the energy taken infinitesimally small, our calculation includes contributions to the band conductivity only from propagating modes. (By propagating modes, we mean travelling wave states, with a real wavevector.) If we use a larger imaginary component to the energy, then the calculation of the conductivity includes contributions from evanescent waves. The formula used for the band conductivity, is not strictly correct with a finite imaginary component to the energy [13] but is sufficient to indicate qualitatively what effects should be observable in the band conductivity. The calculation of the density of states is correct with a finite imaginary component to the energy.

All the calculations are done for magnetic fields with a multiple of $\frac{1}{5}$ of a flux quantum per unit cell, and with a fixed unit cell for the calculation of 5 by 1 units of the antidot potential. We do not take advantage of the reduction in possible unit cell size for the case where there is an integer number of flux quanta to each unit cell of the antidot potential, to avoid introducing any periodic numerical artifacts into the calculation. The spectrum of a periodic system in a magnetic field has long been known to depend on whether the magnetic

field has a rational or an irrational number of flux quanta per unit cell. Because of the inevitable presence of scattering and imperfections, the physical properties of any system are however expected to vary continuously with the magnetic field, so that a numerical calculation, which is restricted to magnetic fields with a rational number of flux quanta per unit cell, can give information about the behaviour of the system in a general magnetic field.

3. Hard-wall antidots—density of states

We consider in this section a hard-wall potential, with a potential $V = 0$ outside circles of radius r_d arranged on a square lattice (see inset to figure 1). A lattice period a of 300 nm is used throughout this paper.

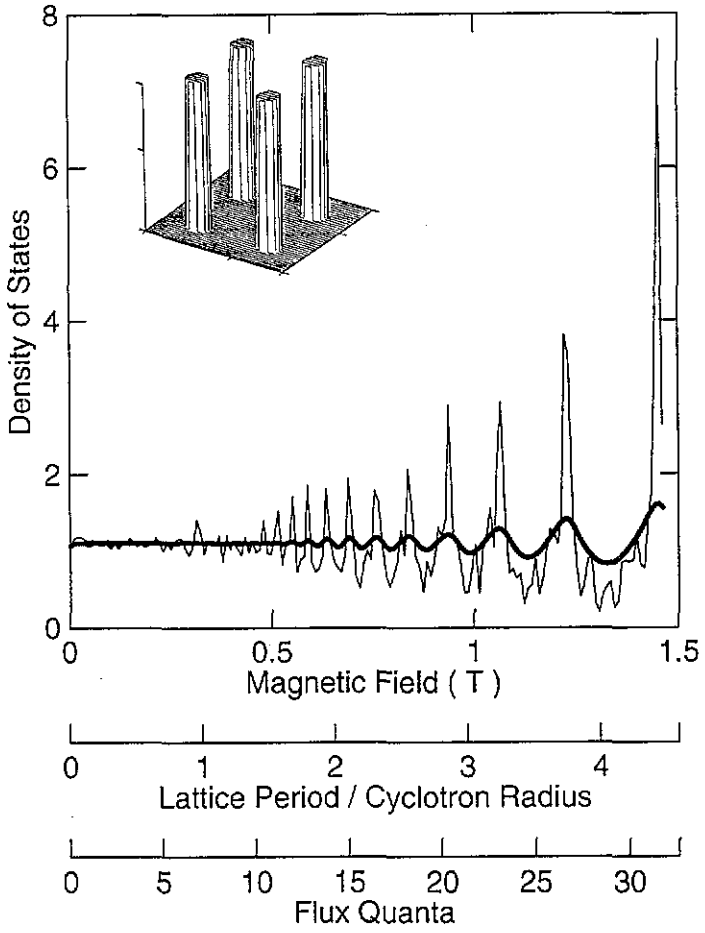

Figure 1. Density of states for an antidot lattice with $r_d/a = 0.15$ and $E = 12.6$ meV. For the thin line, $\mathrm{Im}(E) = 0.05$ meV. and for the thick line $\mathrm{Im}(E) = 0.4$ meV. The form of the potential is shown in the inset.

Figure 1 shows the density of states for a lattice with $r_d/a = 0.2$. The results have been calculated at a fixed energy of 12.6 meV, and with a magnetic field of from 0 to 32 flux quanta per unit cell, equivalent to a maximum field of 1.47 T. For the longer scattering time of 6.5 ps the magnetic field resolution is insufficiently fine to resolve all the structure in the density of states. For the shorter scattering time of 0.81 ps, equivalent to an imaginary component to the complex energy of 0.4 meV, the magnetic field resolution is adequate to resolve all the structure that survives in the density of states.

At the top end of the magnetic field range considered, the density of states shows oscillations due to the formation of Landau levels, which continue to lower magnetic fields, until the cyclotron radius is approximately $0.7a$. Unperturbed cyclotron orbits cannot exist for smaller magnetic fields, with the exception of a relatively small number of orbits enclosing for example four antidots [5]. With the shorter scattering time the energy separation of the Landau levels is sufficiently small for this magnetic field that the Landau level structure has already disappeared at a higher magnetic field. At a lower magnetic field than the main peak of each Landau level, there is some fine structure which is associated with edge states circulating round the antidots.

For larger antidots the individual Landau levels can no longer be observed. Figure 2 shows the density of states for a lattice with $r_d/a = 0.3$. The inset shows the Fourier transform of the density of states calculated over the range from 10 to 20 flux quanta per unit cell and with $\tau_q = 0.8$ ps. At the shorter scattering time we see approximately periodic oscillations in the density of states with a period of approximately $\Delta B = 0.06$ T corresponding to one flux quantum through an area of $0.79a^2$, which matches the highest peak in the Fourier transform. This is the area of the cyclotron orbit for $r_c = a/2$. Note that there is no reason why the oscillations in the density of states should be exactly periodic, since the area enclosed by the cyclotron orbits is varying as the magnetic field changes.

We have examined the local density of states (that is the density of states as a function of position) at the energy and magnetic field of some of the peaks in the density of states. There is no sign in the local density of states of spatial features associated with short, periodic orbits. This is not unreasonable: the effect of the short, periodic orbits is to provide variations in the density of states which are correlated at different energies and magnetic fields, and not necessarily to dominate the density of states at any particular energy. Also for these hard-wall potentials the degenerate cyclotron orbits dominate the oscillations in the density of states, so that there is little spatial modulation of the density of states.

4. Hard-wall antidots—band conductivity

We now consider the variations in the band conductivity. Figure 3 shows the band conductivity calculated with an infinitesimal imaginary component to the energy. (That is there is a scattering time which determines the magnitude of the transport, but there is no energy smoothing, and no cut-off to the path lengths contributing to the band structure.)

We note two features, which also appear for other sizes of antidot and energy. The first is a pronounced oscillation with a period of one flux quantum per unit cell. The inset to figure 3 shows how these oscillations are aligned with a maximum conductivity for an integer number of flux quanta per unit cell. The second is the envelope of the oscillations which shows a pronounced minimum for a cyclotron radius of around $r_c = a/2$ before showing a peak, and falling off again as the magnetic field is increased further, and the cyclotron radius becomes smaller than the separation between the dots, so that the electrons are effectively localized.

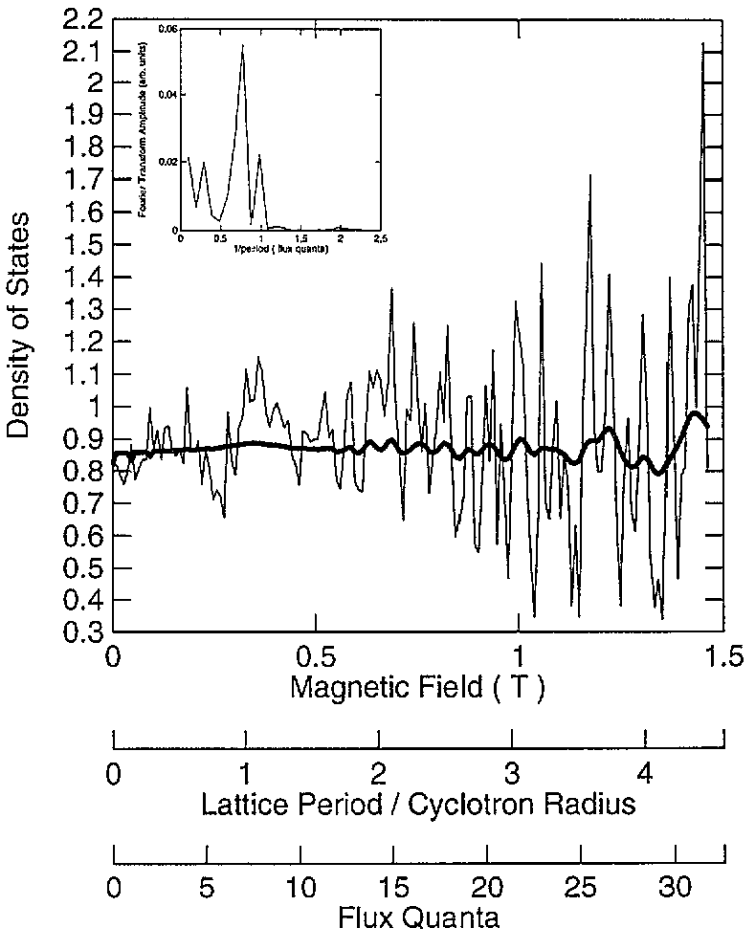

Figure 2. Density of states for an antidot lattice with $r_d/a = 0.30$ and $E = 12.6$ meV. For the thin line, $\mathrm{Im}(E) = 0.05$ meV. and for the thick line $\mathrm{Im}(E) = 0.4$ meV. A Fourier transform of the data between approximately $B = 0.5$ T and $B = 1$ T is shown in the inset.

The periodic oscillation is a manifestation of Hofstadter's butterfly [14]. For a magnetic field with a non-integral number of flux quanta per unit cell there are extra gaps in the band, reducing the group velocity, and the conductivity. For different energies the heights of the peaks vary, and there can be peaks in the conductivity for non-integral numbers of flux quanta. However, since the peaks in the conductivity are tied to the number of flux quanta per unit cell, rather than merely being periodic, energy smoothing should not destroy this structure. However, while energy smoothing will not destroy the oscillations, a level of scattering appropriate for present experimental structures does.

The general behaviour of the band conductivity with a dip in the conductivity before a rise, and a final falling off as the magnetic field is increased is reminiscent of the behaviour in Weiss's experiments [5]. Note that the results here are for the conductivity rather than the resistivity, so that there is no contribution from the Hall effect in our results, whereas in the experiments the resistance falls at high magnetic fields due to the Hall effect.

Figure 4 shows the band-conductivity calculated with a finite imaginary component to the energy corresponding to a scattering time of $\tau_q = 6.5$ ps equivalent to a length of

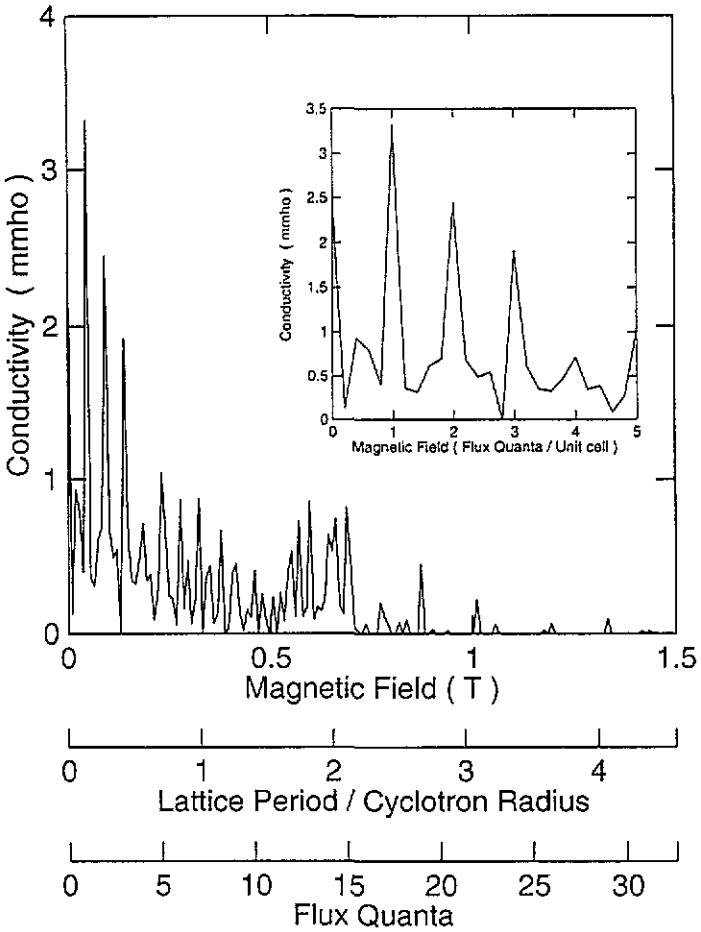

Figure 3. Band conductivity for an antidot lattice with $r_d/a = 0.25$ and $E = 12.6$ meV. Shown in the inset is an expanded view of the data for small magnetic fields.

$v_F\tau = 1600$ nm, where v_F is the group velocity at the Fermi energy. Although this is significantly larger than the lattice period of 300 nm, there is no sign left of the periodic oscillations. Note that oscillations in the density of states with a smaller flux periodicity than the Hofstadter oscillations can still be seen with this scattering time. The Hofstadter's butterfly structure is more sensitive to the scattering because there are in general no periodic classical paths enclosing integer numbers of flux quanta creating the structure, rather the structure reflects the long-range periodicity of the system. We note too that the inclusion of a finite scattering time in the band conductivity calculation leads to a conductivity that reduces steadily as the magnetic field is increased, the dip in the conductivity around $r_c = a/2$ does not survive.

5. Soft-potential antidots

The results for the hard-wall potentials have shown oscillations in the density of states comparable to those reflected in the resistance seen experimentally. The roughly periodic

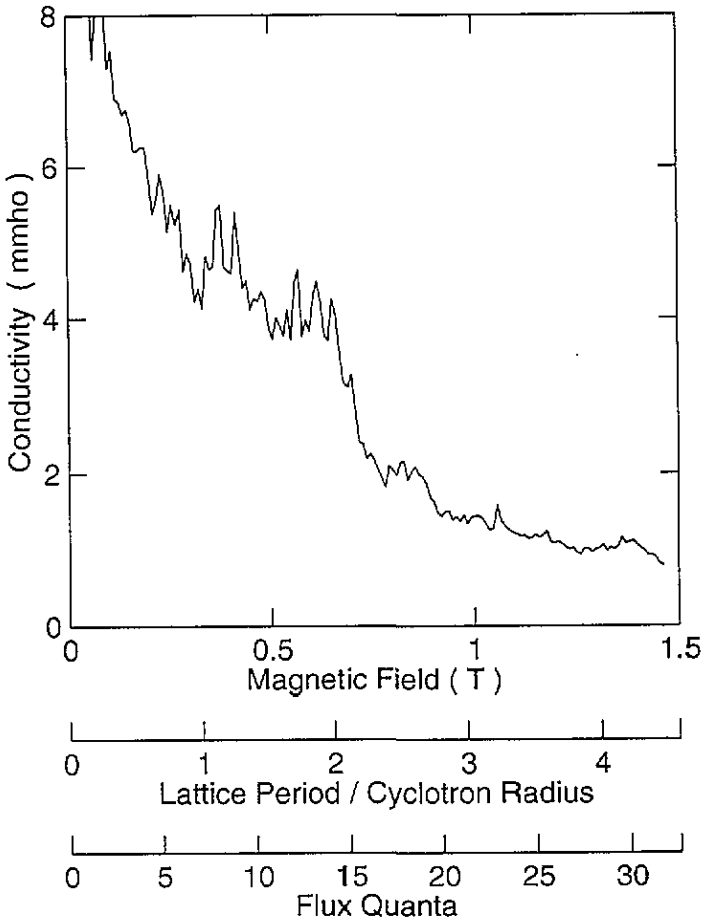

Figure 4. Band conductivity for an antidot lattice with $r_d/a = 0.25$ and $E = 12.6$ meV. An imaginary component to the energy of 0.05 meV has been included in the calculation, corresponding to a quantum scattering time of 6.5 ps.

oscillations in the density of states appear as a continuation of the Landau level structure in the region where the cyclotron radius is about half the lattice spacing. Experimentally a softer potential is more realistic, and will remove the degeneracy between the cyclotron orbits. We have calculated results for some of the potentials used by Weiss *et al* [1] for their semi-classical calculations. We show results here for an electrostatic potential given by $V(x, y) = V_0 \sin^2(\pi x/a) \sin^2(\pi y/a)$, where $V_0 = 15.2$ meV. We use an energy of $E_F = 10$ meV, which gives a dot whose radius (defined as $V(r, 0) = E_F$) is $0.2a$.

Figure 5 shows the results for the density of states. The potential is strong enough to have hidden all direct signs of the Landau levels, whose spacing at the highest magnetic field considered is $\hbar\omega_c = 2.6$ meV. The inset shows the Fourier transform in the region from 10 to 20 flux quanta. The peak of the Fourier transform at a reciprocal period of 0.64 corresponds to an area slightly smaller than an unperturbed cyclotron orbit, and is close to the period of 0.6 we find numerically for a closed orbit going round a maximum of the potential. This is an orbit labelled by 'b' in Weiss's paper [1]. As for the hard-wall case we see no particular signs of these periodic orbits in the local density of states taken at

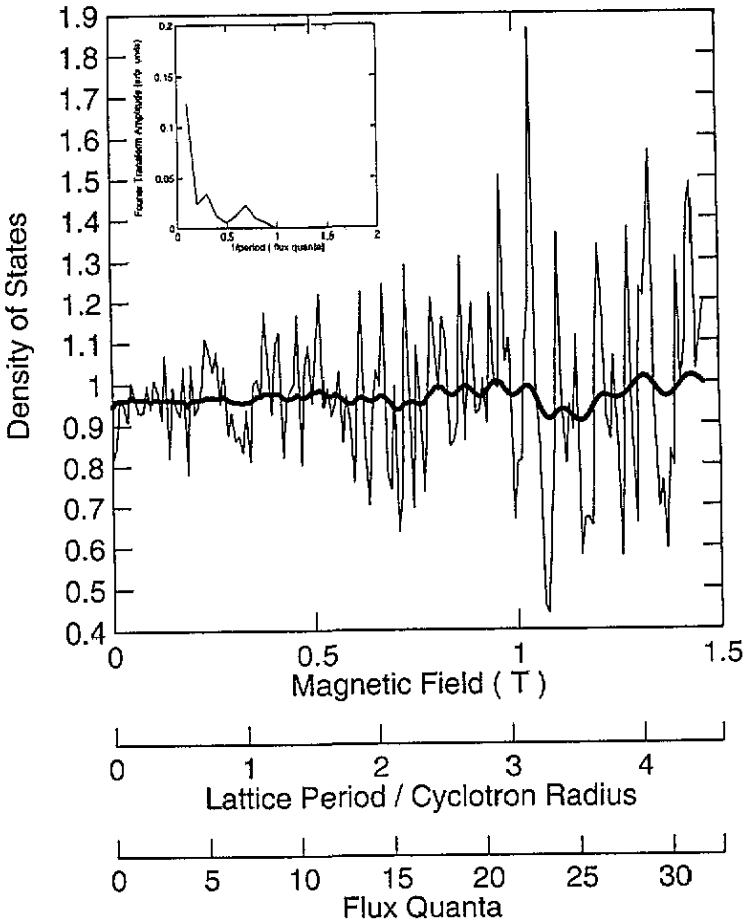

Figure 5. Density of states for a soft-wall lattice. The parameters of the potential are described in the text. For the thin line, $\mathrm{Im}(E) = 0.05$ meV. and for the thick line $\mathrm{Im}(E) = 0.4$ meV. The inset shows the Fourier transform of the thick line in the range from 10 to 20 flux quanta per unit cell.

particular maxima in the density of states.

The behaviour of the band conductivity is shown in figure 6. The Hofstadter's butterfly oscillations are apparent, as for the hard-wall case. The inset shows the Fourier transform of the data. Calculations including a finite imaginary component to the complex energy show that, as for the hard-wall lattice, this periodic structure is suppressed by scattering much faster than the oscillations in the density of states.

6. Summary

We have presented results for the density of states and the band-conductivity of lateral surface superlattices in magnetic fields for the case of a strong modulating potential. The results confirm the existence in the energy-averaged density of states of approximately periodic oscillations associated with the (perturbed) cyclotron orbits. Calculations of the band conductivity with an infinitesimal imaginary component to the energy show strong

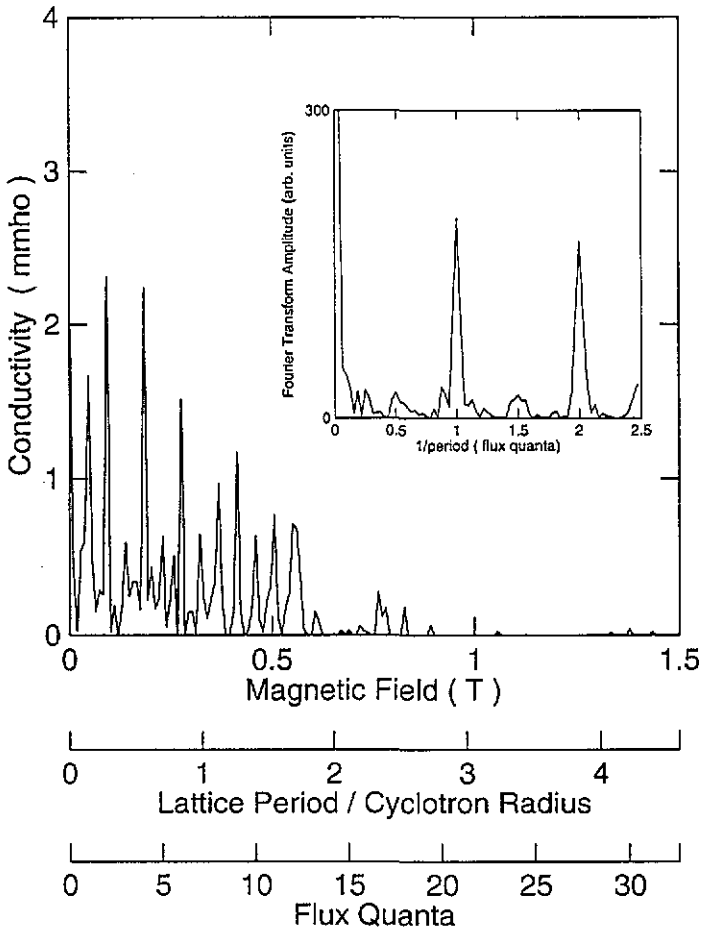

Figure 6. Band conductivity of the soft-wall lattice described in the text. The inset shows the Fourier transform of the data.

periodic oscillations with a period of one flux quantum to each unit cell of the potential, which we associate with the extra splittings introduced into the band-structure when there is a non-integer number of flux quanta per unit cell. Although this structure has a similar flux periodicity to the energy-smoothed density of states, it is suppressed by an imaginary component to the energy, representing the effects of both inelastic scattering and energy averaging, and is not likely to be seen experimentally for structures available at the moment.

The results for the density of states include an energy smoothing equivalent to 0.4 meV. This is much larger than typical estimates of the potential due to random donors [15] and suggests long-range fluctuations in the device properties, or possibly enhanced disorder at the edge of the antidots where there are few electrons to screen the disorder, may be important. Results for smaller arrays should show more structure if long-range fluctuations are important, although edge effects will become important as seen in Kirczenow's calculations [16].

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[11] The calculation is done as a superposition of one-dimensional calculations. With no scattering time put into the calculation the singularities in the one-dimensional densities of states obscure the two-dimensional density of states. All the calculations of densities of states therefore include the Lorentz broadening corresponding to a scattering time of τ_q.

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[13] The band conductivity is given by equations (21) and (22) of paper I. While the long-range propagation of the electrons is described by $\mathrm{Im}(E)/\mathrm{Im}(k)$ even for finite $\mathrm{Im}(E)$, the density of states is not given by $\mathrm{Im}(k)/\mathrm{Im}(E)$ but by $\partial\,\mathrm{Im}(k)/\partial\,\mathrm{Im}(E)$. While $\partial\left(\sum \mathrm{Im}(k)\right)/\partial\,\mathrm{Im}(E)$ can be used quite easily to calculate the overall density of states, as described in paper I, for a conductivity calculation it would be necessary to find the derivatives of individual eigenvalues. We have not implemented this since we are also neglecting other factors which will become important as $\mathrm{Im}(E)$ is increased, in particular the short-range motion of the electron which gives the scattering contribution to the conductivity.

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